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# Observer-based fault detection for networked control systems with network Quality of Services

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ABSTRACT

This paper addresses the problem of the fault detection for linear time-invariant systems over data networks with limited network Quality of Services (QoS). An integrated index $\eta_k$, which related with data dropout, network-induced delay and error sequence, is presented to described the non-ideal QoS, the probabilistic switching between different $\eta_k$ is assumed to obey a homogeneous Markovian chain. Then by view of the augmented matrices approach, the fault detection error dynamic systems are transferred to Markov jumping systems (MJSs). With the developed model and using the bounded real lemma (BRL) for MJSs, an $H_\infty$ observer-based fault detection filter is established in terms of linear matrix inequalities (LMIs) to guarantee that the error between the residual and the weighted faults is made as small as possible. A simulation example is provided to show the effectiveness of the present methods.



## 1. Introduction

Over the last decade a lot of interest has been devoted to Fault detection and isolation (FDI) technology due to an increasing demand for higher performance, higher safety and reliability standards. Fruitful theoretical results and increasing applications in industrial practice, such as in power plant coal mills and robotic systems can be found in [1–9]. Furthermore, with the rapid development of communication networks, recently, a great amount of effort has been devoting to the problems of FDI for networked control systems [10–15].

Data communication networks are used in control systems to interconnect various system devices and components. Such control systems are commonly referred to as Networked control systems (NCSs) [16,17], in which measurement and control signals are transmitted over data networks. NCSs have many advantages over conventional control systems. However, there are also challenges in NCS analysis and design, for instance, time-varying network-induced delay and packet dropout [18–20]. The study on some kind of NCSs has received much attentions recently, the representative works in this field can be seen in [21–26].

Considering a networked control system, the existence of unknown input and network-induced delay may seriously affect the performance of observer-based FDI systems. Recent advances in fault detection over data network include He et al. [12], Mao et al. [11], and Fang et al. [14], and so on. Such as, Mao et al. utilized the multi-rate sampling method together with the augmented state matrix method to model the long random delay networked control system as MJSs, then based on the Riccati equation to have the robustness analysis and $H_\infty$ filter design [11]. Considering the free multiple state delays system, He et al. studied the fault detection problem for a class of discrete-time networked system with both the random

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communication delays and the stochastic date missing, with the help of bounded real lemma (BRL) of MJSs [27,28], a sufficient condition for the existence of the desired fault detection filter is established in terms of a set of LMIs [12]; Fang et al. employed a Takagi–Sugeno (T–S) model to represent an NCS with different network-induced delays, then developed parity-equation and fuzzy-observer-based approaches for fault detection of an NCS [14], respectively. Regardless of these recent developments, to the best of authors' knowledge, the FD problems for NCSs with non-ideal QoS, such as network-induced delay and data dropout described by a unified framework have not been fully investigated yet, which are not only theoretically interesting and challenging, but also very important in practical engineering applications, and motivates our present study.

This paper addresses the problem of the fault detection for linear time-invariant systems over data networks with limited network QoS. The systems under consideration are assumed to be stabilizable. For theoretical development of the fault detection over data networks, a new Markovian jumping system (MJS) model will be developed that combines Souza's idea for MJSs [28] and our recent idea for network modeling [24,26], which allows us to describe both fault detection and NCSs in a unified framework. With the developed model and using the BRL for MJSs [27,28], an $H_\infty$ fault detection filter is established in terms of linear matrix inequalities (LMIs) to guarantee a convergent error dynamics if there is no fault, and to make the effect of disturbances on the residual satisfy a prescribed $H_\infty$ performance. A simulation example is provided to show the effectiveness of the present methods.

The paper is organized as follows. Section 2 proposes a mathematical model to describe both fault detection and NCSs in a unified framework. In Section 3, an $H_\infty$ fault detection filter is designed with consideration of limited network QoS. Numerical example is given in Section 4 to demonstrate the effectiveness of the proposed method. Section 5 concludes the paper.

*Notation:* Throughout the paper, $\mathbb{N}$ stands for positive integers, $\mathbb{R}^n$ denotes the $n$-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the set of $n \times m$ real matrices, $I$ is the identity matrix of appropriate dimensions. The notation $X > 0$ (respectively, $X \geqslant 0$), for $X \in \mathbb{R}^{n\times n}$ means that the matrix $X$ is a real symmetric positive definite (respectively, positive semi-definite).

## 2. Problem statement

Consider the controlled process is a linear time-invariant system

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Ef(t) + D_1\omega(t) \\ y(t) = Cx(t) + D_2\omega(t) \end{cases}, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, $y(t) \in \mathbb{R}^r$ is the measurement output, the unknown input $\omega(t) \in \mathbb{R}^d$ includes model uncertainties and external plant disturbances, and $\omega(t)$ belongs to $L_2 \in [0,\infty)$. $f(t) \in \mathbb{R}^q$ is the fault to be detected. $A,\ B,\ C,\ E,\ D_1$ and $D_2$ are constant matrices with appropriate dimensions, $C$ is of full row rank.

The following assumptions, which are common in NCS research reported in the open literature [26,29,30], are also made in this work:

(1) The sensors are clock-driven, the controller and actuators are event-driven;
(2) Data, either from measurement or for control, is transmitted with a single-packet, and full state variables are available for measurements;
(3) The real input $u(t)$, realized through a zero-order hold, is a piecewise constant function. For example, the $u(t)$ is keep constant in zero-order holding period in Fig. 1.
(4) In case of non-ordered sequences, the time stamping technique is applied to choose the latest message. Therefore, the control signal is chosen as the solid line in Fig. 1.

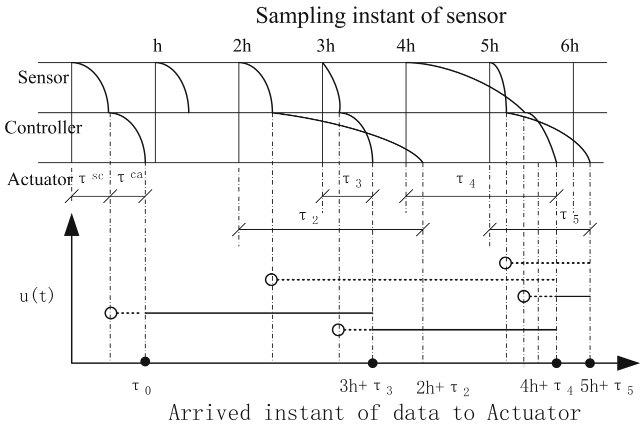


**Fig. 1.** Time diagram for data transmission.

It is worth mentioning that the sampling period of a sensor is prescribed for control algorithm design, and thus the sensor can be assumed to be clock-driven. However, an actuator does not change its output to the plant under control until an updated control signal is received, implying that the actuator is event-driven.

From the above assumptions, using a similar modeling technique in [26,29,30], we model the real system (1) controlled over network as

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Ef(t) + D_1\omega(t), \quad t \in [i_kT + \tau_{i_k}, i_{k+1}T + \tau_{i_{k+1}}) \\ y(t) = Cx(t) + D_2\omega(t) \\ u(t^+) = u(t - \tau_{i_k}), \quad t \in \{i_kT + \tau_{i_k}, \quad k = 1, 2, \ldots\} \end{cases}, \tag{2}$$

where $u(t^+) = \lim_{\hat{t} \to t+0} u(\hat{t})$, $T$ denotes the sampling period, $i_k$ $(k = 1, 2, 3, \ldots)$ are some integers such that $\{i_1, i_2, i_3, \ldots\} \subset \{0, 1, 2, 3, \ldots\}$. $\tau_{i_k}$ is the time from the instant $i_kT$ when sensors sample from the plant to the instant when actuators send control actions to the plant. Here, we have assumed that the control computation and other overhead delays are included in $\tau_{i_k}$. Simultaneously, we choose the present arrive actuator instant $i_kT + \tau_{i_k}$ and next arrive actuator instant $i_{k+1}T + \tau_{i_{k+1}}$ as duration of zero-order hold, Obviously, $\cup_{k=1}^{\infty}[i_kT + \tau_{i_k}, i_{k+1}T + \tau_{i_{k+1}}] = [t_0, \ \infty)$, $t_0 \geqslant 0$. Then the system (2) can be rewritten as

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(i_kT) + Ef(t) + D_1\omega(t), \quad t \in [i_kT + \tau_{i_k}, i_{k+1}T + \tau_{i_{k+1}}) \\ y(t) = Cx(t) + D_2\omega(t) \end{cases}. \tag{3}$$

Since $u(i_kT) = u[t - (t - i_kT)]$, define $\eta(t) = t - i_kT$, $k = 1, 2, 3, \ldots$ in every internal $[i_kT + \tau_{i_k}, \ i_{k+1}T + \tau_{i_{k+1}})$. Then Eq. (3) becomes

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t - \eta(t)) + Ef(t) + D_1\omega(t), \quad t \in [i_kT + \tau_{i_k}, i_{k+1}T + \tau_{i_{k+1}}) \\ y(t) = Cx(t) + D_2\omega(t) \end{cases}. \tag{4}$$

From the definition of $\eta(t)$, it can be seen that $\eta(t)$ is discontinuous at the points $i_kT + \tau_{i_k}$, $k = 1, 2, 3, \ldots$. If $i_{k+1} - i_k$ and $\tau_{i_k}$ have upper bound, then $\eta(t)$ is a bounded piecewise continuous function. Therefore, in some sense, the NCS (3) is equivalent to a linear system with a bounded time-varying delay.

**Remark 1.** In (2), $\{i_1, i_2, i_3, \ldots\}$ is a subset of $\{0, 1, 2, 3, \ldots\}$. Moreover, it is not required that $i_{k+1} > i_k$. If $|(i_{k+1} - i_k)| = 1$ it means no data packet dropout in the transmission. If $i_{k+1} > i_k + 1$, it means that some data packet dropout and no non-ordered sequence occurs. If $i_{k+1} < i_k + 1$, it means that non-ordered sequence occurs, which includes $\tau_k = \tau_0$ and $\tau_k < h$ as the special cases. It is shown in Fig. 1. Therefore, (2) can be viewed as a general form of NCS, where the effect of the networked-induced delay, data packet dropout and non-ordered sequence are simultaneously considered [26,30].

From the above definition of $\eta(t)$, we have $\eta(t) \leqslant sup[(i_{k+1} - i_k)T + \tau_{i_{k+1}}] \leqslant hT$ ($h$ is a positive integer). Then similar to the discrete operation in Hu and Zhu [23], integration of (4) over a sampling interval $[kT; \ (k+1)T)$ yields

$$x_{k+1} = A_s x_k + \sum_{i=0}^{h} B_i^k u_{k-i} + \overline{E} f_k + \overline{D}_1 \omega_k, \tag{5}$$

where $A_s = e^{AT}$, $B_i^k = \int_{t_{i-1}^k}^{t_i^k} e^{A(T-s)} ds B$, $t_{-1}^k = 0, t_0^k = T$, $\overline{E} = \int_0^T e^{A(T-t)} E dt$, $\overline{D}_1 = \int_0^T e^{A(T-t)} D_1 dt$.

Assume that the filter is embedded in the system through the communication network. For simplify, which is said as network-based filter. In NCS, since the signal transmitted over communication network, there have the number of $i$, $i \in \{0, 1, \ldots, h\}$ possible network-induced delay. Therefore, the system output $y_k^i$ obtained in network-based filter can be expressed as

$$y_k^i = \sum_{i=0}^{h} \delta(\eta_k, i) C x_{k-i} + D_2 \omega_k, \tag{6}$$

where $\delta(\cdot, \cdot)$ stands for the Kronecker delta, i.e.

$$\delta(j, i) = \begin{cases} 0 & \text{if } j \neq i \\ 1 & \text{if } j = i \end{cases}. \tag{7}$$

$\eta_k$ is a stochastic variable whose role is to determine the size of the occurred $\eta(t)$ at time $k$. Similar to [11,12,23], we assume that $[\eta_k]$ is a discrete-time homogeneous Markov chain taking values in the finite state space $\Xi := \{0, 1, \ldots, h\}$ and the stationary transition probability matrix $\Lambda = [\lambda_{ij}]$, where $\lambda_{ij} := \text{Prob}\{\eta_{k+1} = j \mid \eta_k = i.\}$ and $0 \leqslant \lambda_{ij} \leqslant 1$, $\sum_{j=0}^{m} \lambda_{ij} = 1$.

For the convenient development, setting the augmented state vector of the plant be $z_k = [x_k^T, \ldots, x_{k-h}^T, u_{k-1}^T, \ldots, u_{k-h}^T]^T$, and combining Eqs. (5) and (6) into a unified framework, we obtain

$$\begin{cases} z_{k+1} = \widetilde{A} z_k + \widetilde{B} u_k + \widetilde{E} f_k + \widetilde{D}_1 \omega_k \\ y_k^i = \widetilde{C}_i z_k + D_2 \omega_k \end{cases}, \tag{8}$$

where

$$\widetilde{A}=\begin{bmatrix}[A_s,\underbrace{0_{n\times n},\ldots,0_{n\times n}}_{h}] & [B_1^k,\ldots,B_h^k] \\ \begin{bmatrix} I_{n\times n} & \cdots & 0 & 0_{n\times n} \\ 0 & \ddots & \vdots & 0_{n\times n} \\ 0 & \cdots & I_{n\times n} & 0_{n\times n}\end{bmatrix}_{h\times(h+1)} & \begin{bmatrix} 0_{n\times m} & \cdots & 0 \\ 0 & \ddots & \vdots \\ 0 & \cdots & 0_{n\times m}\end{bmatrix}_{h\times h} \\ [\underbrace{0_{m\times n},\ldots,0_{m\times n}}_{h+1}] & [0_{m\times m} \quad \cdots \quad 0]_{h\times h} \\ \begin{bmatrix} 0_{m\times n} & \cdots & 0 \\ 0 & \ddots & \vdots \\ 0 & \cdots & 0_{m\times n}\end{bmatrix}_{(h-1)\times(h+1)} & \begin{bmatrix} I_{m\times m} & \cdots & 0 \\ 0 & \ddots & \vdots \\ 0 & \cdots & I_{m\times m}\end{bmatrix}_{(h-1)\times h}\end{bmatrix},$$

$$\widetilde{B}=\begin{bmatrix} B_0^k \\ 0_{hn\times m} \\ 0_{m\times m} \\ 0_{(h-1)m\times m}\end{bmatrix},\quad \widetilde{E}=\begin{bmatrix} \overline{E} \\ 0_{hn\times m} \\ 0_{m\times m} \\ 0_{(h-1)m\times m}\end{bmatrix},\quad \widetilde{D}_1=\begin{bmatrix} \overline{D}_1 \\ 0_{hn\times m} \\ 0_{m\times m} \\ 0_{(h-1)m\times m}\end{bmatrix},$$

$$\widetilde{C}_i=\left[\,[\delta(\eta_k,0)C,\ldots,\delta(\eta_k,i)C,\ldots,\delta(\eta_k,h)C],\quad [\underbrace{0_{m\times m},\ldots,0_{m\times m}}_{h}]\,\right].$$

The main purpose of this paper is to investigate the problem of fault detection for NCSs with non-ideal network QoS, which includes the residual generation and a residual evaluation stage. Now, we are in a position to achieve this objective coupled with the above NCS augmented model (8).

The network-based filter of the following Luenberger observer form is sought:

$$\begin{cases} \hat{z}_{k+1}=\widetilde{A}\hat{z}_k+\widetilde{B}u_k+K_i(y_k^i-\hat{y}_k) \\ \hat{y}_k=\widetilde{C}_i\hat{z}_k \end{cases},\tag{9}$$

where $\hat{z}_k$ is the augmented state estimation vector, $K_i$ is the filter's gain to be designed.

Set the filter error be $e_k=z_k-\hat{z}_k$, and the residual vector $\varepsilon_k$ be $S(y_k^i-\hat{y}_k)$, where $S$ is a suitable weighting matrix designed to assure isolability properties. Then the overall fault detection dynamics is governed by the following Markov jumping systems (MJSs):

$$\begin{cases} e_{k+1}=\overline{A}_ie_k+\overline{B}_i\upsilon_k \\ \varepsilon_k=\overline{C}_ie_k+\overline{D}\upsilon_k \end{cases},\tag{10}$$

where

$$\overline{A}_i=(\widetilde{A}_i+K_i\widetilde{C}_i),\quad \overline{B}_i=\left[K_iD_2+\widetilde{D}_1,\widetilde{E}\right],$$

$$\overline{C}_i=S\widetilde{C}_i,\quad \overline{D}=[SD_2 \quad 0],\quad \upsilon_k=[\omega_k \quad f_k]^T.$$

In order to reduce the false alarm rate, we also use threshold selector to evaluate the residual for the residual generator. The following 2-norm of residual signal is chosen as residual evaluation function as that in [3].

$$J_{th}=\sup_{\omega\in l_2,f=0}\mathbb{E}\left\{\left\{\sum_{s=0}^{L}\varepsilon_s^T\varepsilon_s\right\}^{1/2}\right\},\tag{11}$$

where $L$ denotes the maximum time step of the evaluation function.

Then, the decision is made based on the following rule:

$$\begin{cases} J(k)>J_{th}\Rightarrow \text{ A fault is detected} \\ J(k)\leqslant J_{th}\Rightarrow \text{ No faults} \end{cases}.\tag{12}$$

**Remark 2.** In this paper, we only consider the observer-based network fault detection filter design problem, and assume that the control input signal $u_k$ in (9) can be directly obtained from the control input in (8). However, if the control signal $u_k$ is transmitted over network, then the $u_k$ should be replaced by $\sum_{i=0}^{h}\delta(\eta_k,i)u_{k-i}$ in (9) due to the negative effect of the integrated delay $\eta_k$.

After the above manipulations, we are now attempting to formulate some practically computable criteria to obtain the filter gain $K_i$ described by (9). The following definition is useful in deriving the criteria.

**Definition 1.** For the given filter gain matrix $K_i$, system (10) is said to be mean-square stable with an $H_\infty$ norm bound $\gamma$, if the following hold:

(1) For system (10) with $\upsilon_k \equiv 0$, is said to be mean-square stable if the following holds [28]:

$$\mathbb{E}\left\{\sum_{k=0}^{\infty} e_k^T e_k\right\} \rightarrow 0, \quad \text{as } k \rightarrow \infty \tag{13}$$

for any initial condition $e_0$ and initial distribution $\eta_0 \in \Xi$.

(2) For a given scalar $\gamma > 0$, the following inequality holds for any non-zero $\upsilon_k$

$$\mathbb{E}\left\{\sum_{k=0}^{\infty} \varepsilon_k^T \varepsilon_k\right\} \leqslant \gamma^2 \mathbb{E}\left\{\sum_{k=0}^{\infty} \upsilon_k^T \upsilon_k\right\}. \tag{14}$$

**Remark 3.** Based on performance index (14), the design of fault detection filter (9) is formulated as an $H_\infty$-filtering problem, which can be solved by using optimization technique, and will be shown in the following section.

## 3. Main results

To finish the filter design based on the MJSs model (10), we first introduce a lemma concerning the norm of discrete-time Markovian jump linear system [27,28].

**Lemma 1.** *Given a scalar $\gamma > 0$ and a stochastic system*

$$\begin{cases} x_{k+1} = A_i x_k + B_i \omega_k \\ z_{k+1} = C_i x_k + D_i \omega_k \end{cases}, \tag{15}$$

*where*

$$A_i = A(\theta(k))|_{\theta(k)=i}, B_i = B(\theta(k))|_{\theta(k)=i},$$

$$C_i = C(\theta(k))|_{\theta(k)=i}, D_i = D(\theta(k))|_{\theta(k)=i},$$

*and $\theta(k)$ is a discrete-time homogeneous markov chain with finite state space $\Xi = \{0, 1, \ldots, h\}$ and stationary transition probability matrix $\Lambda = [\lambda_{ij}]$, where $\lambda_{ij} := Prob\{\lambda(k+1) = j \mid \lambda(k) = i.\}$. Then* the following conditions are equivalent:

(1) *System (15) is internally mean-square stable and its $H_\infty$ norm defined by*

$$\sup_{\omega \epsilon l_2, \omega \neq 0, x(0)=0} \frac{\mathbb{E}[\|z\|_2]}{\|\omega\|_2} \tag{16}$$

*is less than $\gamma$.*

(2) *There exist matrices $P_i$ and $G_i$ satisfying the following LMIs:*

$$\begin{bmatrix} \bar{P}_i & A_i^T G_i^T & 0 & C_i^T \\ G_i A_i & G_i + G_i^T - \overline{P}_i & G_i B_i & 0 \\ 0 & B_i^T G_i^T & \gamma I & D_i^T \\ \overline{C}_i & 0 & D_i & \gamma I \end{bmatrix} > 0, \tag{17}$$

*where*

$$\overline{P}_i = \sum_{j=0}^{h} \lambda_{ij} P_j, i = 0, \ldots, h.$$

Based on above Lemma 1 and the MJSs model (10), we have the following result to determine the filter gain $K_i$.

**Theorem 1.** *Given scalars S, $\gamma > 0$, if there exist symmetric positive definite matrices $P_i$, $G_i > 0$ with appropriate dimensions, matrices $L_i$ with appropriate dimensions, such that the following LMIs hold*

$$\begin{bmatrix} \bar{P}_i & (G_i\widetilde{A}_i + L_i\widetilde{C}_i)^T & 0 & 0 & \overline{C}_i^T \\ (G_i\widetilde{A}_i + L_i\widetilde{C}_i) & G_i + G_i^T - \overline{P}_i & (L_iD_2 + G_i\widetilde{D}_1) & G_i\widetilde{E} & 0 \\ 0 & (D_2^TL_i^T + \widetilde{D}_1^TG_i^T) & \gamma I & 0 & D_2^TS^T \\ 0 & \widetilde{E}^TG_i^T & 0 & \gamma I & 0 \\ \overline{C}_i & 0 & SD_2 & 0 & \gamma I \end{bmatrix} > 0, \tag{18}$$

*where*

$$\overline{P}_i = \sum_{j=0}^{h} \lambda_{ij}P_j, \quad i = 0, \ldots, h. \tag{19}$$

*$\widetilde{A}_i$, $\widetilde{C}_i$, $\widetilde{E}$, $\widetilde{D}_1$, $D_2$ and $\overline{C}_i$ are defined in (8) and (10), then system (10) is mean-square stable with its $H_\infty$ norm being less than $\gamma$ and $K_i = G_i^{-1}L_i$.*

**Proof 1.** Define $L_i = G_iK_i$, then based on Lemma 1 and MJS model (10), it is direct to derive the Eq. (18), the detailed process is omitted here. This completes the proof. □

So far the filter has been designed to satisfy the Definition 1. Also, the results in Theorem 1 suggest some other types of optimization problems to obtain the minimum noise attenuation level bound $\gamma$. Therefore, an optimal fault detection filter can be readily found by solving the following optimization problem:

(P1) The optimal $H_\infty$ filter design problem

$$\text{Minimize}: \gamma, \quad s.t. \ (18) \text{ and } (19). \tag{20}$$

**Remark 4.** Theorem 1 is derived based on Lemma 1, the main advantage of Theorem 1 is that it transfers the non-ideal network QoS to an integrated delay bound and utilizes the MJSs model to couple the fault detection of NCSs and non-ideal network QoS into a unified framework. Otherwise, the computational complexity of Theorem 1 mainly depends on the range of the finite state space $\Xi := \{0, 1, \ldots, h\}$ of the Markov chain, which can be solved using the efficient LMI toolbox/Matlab.

## 4. Numerical example

Consider the system (1) with the following parameters

$$A = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0.1 \\ 0.05 \end{bmatrix}, \tag{21}$$

$$C = [1 \quad 1], \quad D_2 = 0.1, \quad E = -B. \tag{22}$$

Assume that the sampling period of the NCS is 0.01 s, the state space of the Markov chain is $\Xi := \{0 \quad 1 \quad 2\}$. By some calculation, the Markovian jump system model is obtained as (10) with the following parameters.

$$\widetilde{A} = \begin{bmatrix} 0.9999 & 0.0098 & 0 & 0 & 0 & 0 & 0.0199 & 0.0198 \\ -0.0294 & 0.9606 & 0 & 0 & 0 & 0 & 0.0105 & 0.0115 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$\widetilde{B} = [0.0201 \quad 0.0095 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0]^T,$$

$$\widetilde{E} = [-0.02 \quad -0.0095 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T,$$

$$\widetilde{D}_1 = [0.001 \quad 0.0005 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T,$$

$$\widetilde{C}_0 = [1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0],$$

$$\widetilde{C}_1 = [0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0],$$

$$\widetilde{C}_2 = [0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0].$$

The transition probability matrix is given by [12]

$$\Lambda := \begin{bmatrix} \lambda_{0,0} & \lambda_{0,1} & \lambda_{0,2} \\ \lambda_{1,0} & \lambda_{1,1} & \lambda_{1,2} \\ \lambda_{2,0} & \lambda_{2,1} & \lambda_{2,2} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.4 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}, \tag{23}$$

the initial model is set to be $\tau_0 = 0$. The fault signal $f_k$ is given as

$$f_k = \begin{cases} 1 + 0.5\cos(4\pi k) & \text{for } k = 100, 101, \ldots, 200 \\ 0 & \text{others} \end{cases}, \tag{24}$$

and the unknown input $\omega_k$ is supposed to be a random uniformly distributed over $[-0.5 \quad 0.5]$.

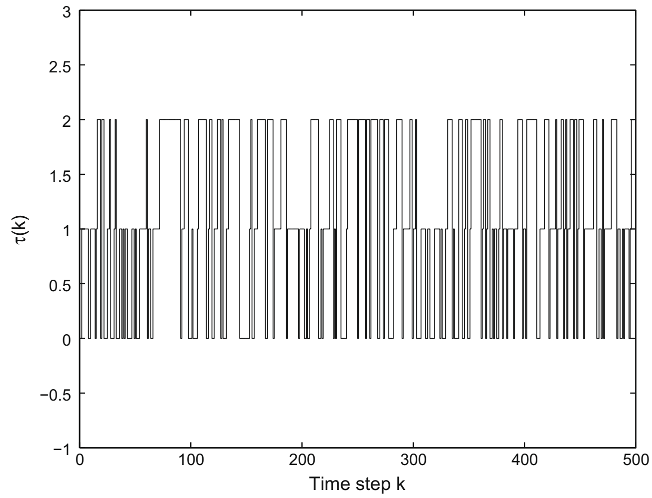


**Fig. 2.** Measurement mode over network.

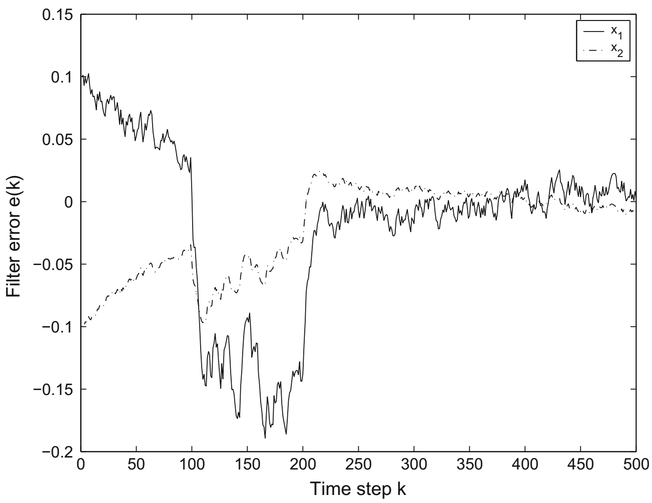


**Fig. 3.** Filter error signal $e(k)$.

With the given parameters and based on the optimal $H_\infty$ filter design problem (P1), we obtain the minimum noise attenuation level bound $\gamma_{opt} = 1.177$ of the fault detection dynamics (10), and the parameters of the fault detection filter in different modes are given by

$$K_0 = \begin{bmatrix} -0.2105 \\ -0.0626 \\ -0.2026 \\ -0.0676 \\ -0.1601 \\ -0.0546 \\ 0.0000 \\ 0.0000 \end{bmatrix}, \quad K_1 = \begin{bmatrix} -0.2081 \\ -0.0602 \\ -0.2001 \\ -0.0649 \\ -0.1975 \\ -0.0728 \\ 0.0000 \\ 0.0000 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -0.1648 \\ -0.0418 \\ -0.1566 \\ -0.0443 \\ -0.1605 \\ -0.0532 \\ 0.0000 \\ 0.0000 \end{bmatrix}. \tag{25}$$

Now, we consider the time-domain simulation using the obtained fault detection filter.

Fig. 2 shows the measurement mode with random delays satisfying Markov transition probability matrix described by (23). $\eta_k = 0, 1, 2$ means that the measurement transmitted over the network ideally and with one-step and two-step delay, respectively. Fig. 3 shows the filter error described by (10). From Fig. 3, it can be seen that the error dynamic system (10)

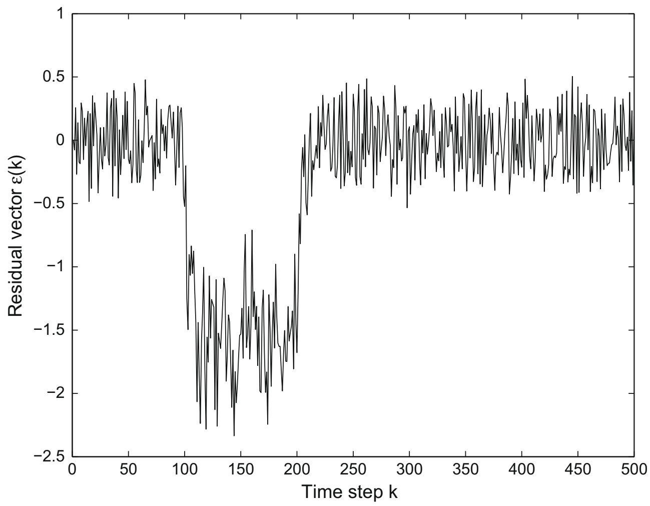


**Fig. 4.** Residual signal $\varepsilon_k$.

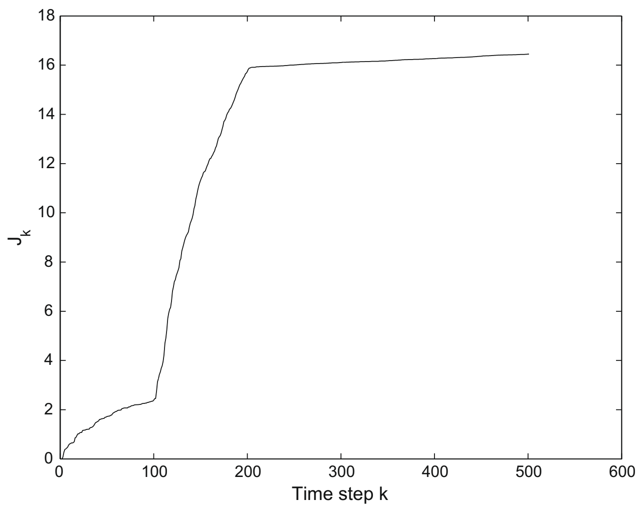


**Fig. 5.** Evolution of $J(k)$.

trends to stable under the external impact of fault signal $f_k$ and disturbance signal $\omega_k$. Fig. 4 shows the generated residual signal $\varepsilon_k$, and the evolution of $J(k) = \left\{\sum_{s=0}^{k}\varepsilon^T(s)\varepsilon(s)\right\}^{1/2}$ is presented in Fig. 5. We select a threshold as $J_{th} = sup_{f=0}\mathbb{E}\left\{\sum_{s=0}^{300}\varepsilon^T(s)\varepsilon(s)\right\}^{1/2}$, after 500 times simulations, we obtain an average value $J_{th} = 3.9304$. From Fig. 5, it can be shown that $3.7901 = J(108) < J_{th} < J(109) = 4.2241$. Therefore, the fault can be detected in nine time steps after its occurrence.

## 5. Conclusion

This paper is concerned with the problem of the fault detection for linear time-invariant systems over data networks with limited network QoS. A new Markovian jumping system (MJS) model has been developed to couple observer-based fault detection filter with our recent idea for network modeling [24,26] in a unified framework. With the developed model and using the bounded real lemma (BRL) for MJSs [27,28], an $H_\infty$ fault detection filter is established in terms of linear matrix inequalities (LMIs) to guarantee the convergent error dynamics if there is no fault in the system, and to make the effect of disturbances on the residual satisfying a pre-described $H_\infty$ performance. Numerical example has been given to demonstrate the effectiveness of the proposed method.

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## References

[1] J. Chen, R.J. Pattron, Robust Model-Based Fault Diagnosis for Dynamic System, Kluwer, London, UK, 1999.
[2] M.Y. Zhong, S.X. Ding, J. Lam, A *LMI* approach to design robust fault detection filter for uncertain LTI systems, Automatica 39 (3) (2003) 543–550.
[3] M.Y. Zhong, H. Ye, P. Shi, Fault detection for markovian jump systems, IET Control Theory Appl. 152 (4) (2005) 397–402.
[4] A. Monteriu, P. Asthana, K. Valavanis, S. Longhi, Experimental validation of a real-time model-based sensor fault detection and isolation system for unmanned ground vehicles, in: Control and Automation, 2006, MED'06, 14th Mediterranean Conference, 2006, pp. 1–8.
[5] A. Monteriu, P. Asthana, K. Valavanis, S. Longhi, Model-based sensor fault detection and isolation system for unmanned ground vehicles: theoretical aspects (part I), in: 2007 IEEE International Conference on Robotics and Automation, 2007, pp. 2736–2743.
[6] S.N. Huang, K.K. Tan, Fault detection, isolation, and accommodation control in robotic systems, IEEE Trans. Automation Sci. Eng. 5 (3) (2008) 480–489.
[7] S.B. Bao Lin Jorgensen, P.F. Odgaard, Observer and data-driven-model-based fault detection in power plant coal mills, IEEE Trans. Energy Convers. 23 (2) (2008) 659–668.
[8] H.M. Gomes, N.R.S. Silva, Some comparisons for damage detection on structures using genetic algorithms and modal sensitivity method, Appl. Math. Model. 32 (11) (2008) 2216–2232.
[9] H.N. Wu, $H_\infty$ fuzzy control design of discrete-time nonlinear active fault-tolerant control systems, Int. J. Robust Nonlinear Control 19 (2009) 1129–1149.
[10] M.Y. Zhong, Q.L. Han, Fault detection filter design for a class of networked control systems, in: Proceedings of the 6th World Congress on Intelligent Control and Automation (WCICA'06), Dalian, China, 21–23 June 2006, pp. 215–219.
[11] Z. Mao, B. Jiang, P. Shi, $H_\infty$ fault detection filter design for networked control systems modelled by discrete markovian jump systems, IET Control Theory Appl. 1 (5) (2007) 1336–1343.
[12] X. He, Z.D. Wang, Y.D. Ji, D.H. Zhou, Network-based fault detection for discrete-time state-delay systems: a new measurement model, Int. J. Adapt. Control Signal Process 22 (2008) 510–528.
[13] P. Zhang, S.X. Ding, P.M. Frank, M. Sader, Fault detection of networked control systems with missing measurements, in: Proceedings of Asian Control Conference, Melbourne, Australia, 2004, pp. 1258–1263.
[14] Y. Zheng, H.J. Fang, Hua O. Wang, Takagi–Sugeno fuzzy-model-based fault detection for networked control systems with markov delays, IEEE Trans. Syst. Man Cybern. – B: Cybern. 36 (4) (2006) 924–929.
[15] H.J. Fang, H. Ye, M.Y. Zhong, Fault diagnosis of networked control systems, Annu. Rev. Control 31 (2007) 55C68.
[16] L.A. Montestruque, P.J. Antsaklis, On the model-based control of networked systems, Automatica 39 (2003) 1837–1843.
[17] C.L. Ma, H.J. Fang, Research on mean square exponential stability of networked control systems with multi-step delay, Appl. Math. Model. 30 (9) (2006) 941–950.
[18] T.C. Yang, Networked control system: a brief survey, IET Control Theory Appl. 153 (4) (2006) 403–412.
[19] Y.-C. Tian, M.O. Tad D. Levy, T. Gu, C. Fidge, Queuing packets in communication networks networked control systems, in: Proceedings of the 6th World Congress on Intelligent Control and Automation (WCICA'06), Dalian, China, 21–23 June 2006, pp. 210–214.
[20] J.P. Hespanha, P. Naghshtabrizi, Yonggang Xu, A survey of recent results in networked control systems, Proc. IEEE 95 (1) (2007) 138–162.
[21] W. Zhang, M.S. Branicky, S.M. Phillips, Stability of networked control systems, IEEE Control Syst. Mag. 21 (2001) 84–99.
[22] D.S. Kim, Y.S. Lee, W.H. Kwon, H.S. Park, Maximum allowable delay bounds of networked control systems, Control Eng. Pract. 11 (2003) 1301–1313.
[23] S.S. Hu, Q.X. Zhu, Stochastic optimal control and analysis of stability of networked control systems with long delay, Automatica 39 (2003) 1877–1884.
[24] D. Yue, Q.L. Han, J. Lam, Network-based robust $H_\infty$ control of systems with uncertainty, Automatica 41 (2005) 999–1007.
[25] F.W. Yang, Z.D. Wang, Y.S. Hung, Mahbub Gani, $H_\infty$ control for networked systems with random communications delays, IEEE Trans. Automation Control 51 (3) (2006) 511–518.
[26] C. Peng, Y.-C. Tian, State feedback controller design of networked control systems with interval time-varying delay and nonlinearity, Int. J. Robust Nonlinear Control 18 (2008) 1285–1301.
[27] Pete Seiler, Raja Sengupta, A bounded real lemma for jump systems, IEEE Trans. Automatic Control 48 (9) (2003) 1651–1654.
[28] C.E. de Souza, A model-independent $H_\infty$ filter design for discrete time markovian jump linear systems, in: Proceedings of the 42nd IEEE Conference on Decision and Control, Maui, Hawaii, USA, December 2003, pp. 2811–2816.
[29] D. Yue, Q.L. Han, C. Peng, State feedback controller design of networked control systems, IEEE Trans. Circuits Systems II: Express Briefs 51 (2004) 640–644.
[30] C. Peng, Y.-C. Tian, Networked $H_\infty$ control of linear systems with state quantization, Inform. Sci. 177 (2007) 5763–5774.